Large-Scale Knowledge Processing

Optimization Techniques (1)
Practice Brief Solutions

p.7 Practice

Let x_1, x_2, and x_3 denote the production amounts of each product.

$$\begin{array}{ll} \text{maximize} & 3x_1 + 5x_2 + 4x_3 \\ \text{subject to} & 4x_1 + 2x_2 + \ \ x_3 \leq 6 \\ & \ \ x_1 + 2x_2 + 4x_3 \leq 7 \\ & 5x_1 + 2x_2 + 3x_3 \leq 9 \\ & 3x_1 + 3x_2 + 2x_3 \leq 8 \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0 \end{array}$$

p.8 Practice

Let x_1, x_2, $\ldots x_n$ denote the production amounts of each product.

$$\begin{array}{lll} \text{maximize} & \displaystyle\sum_{i=1}^{n} a_i x_i & \\ \text{subject to} & \displaystyle\sum_{i=1}^{n} c_{ij} x_i \leq b_j & (j = 1, 2, \ldots, m) \\ & x_i \geq 0 & (i = 1, 2, \ldots, n) \end{array}$$

p.16 Practice

The optimal solution is $(x_1, x_2) = (\frac{4}{3}, \frac{2}{3})$, and the optimal value is $\frac{26}{3}$.

p.18 Practice

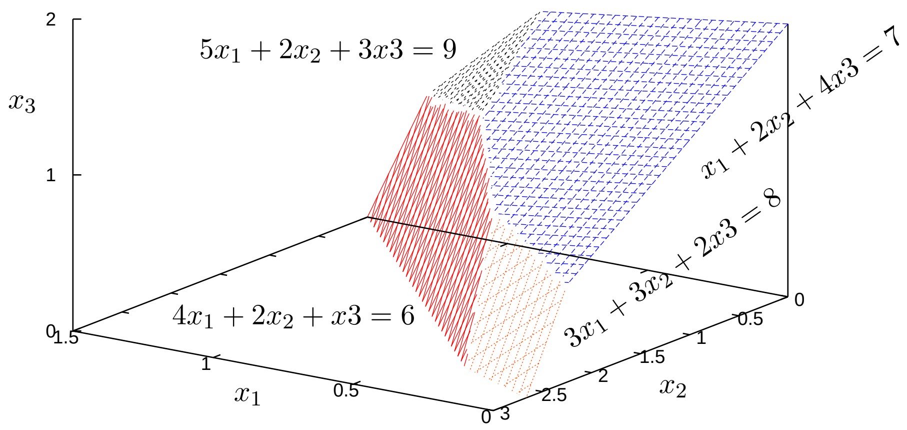
The optimal solution is $(x_1, x_2) = (2, 0)$, and the optimal value is 8.

p.20 Practice

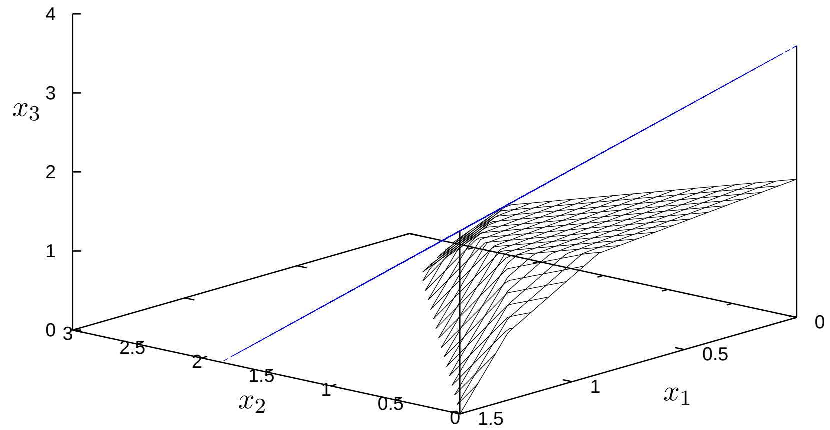
(a) Figure omitted. The optimal solution is $(x_1, x_2) = (\frac{4}{3}, \frac{2}{3})$, and the optimal value is $\frac{26}{3}$.
(b) Figure omitted. The optimal solution is $(x_1, x_2) = (0, 0)$, and the optimal value is 0.
(c) Figure omitted. The optimal solution is $(x_1, x_2) = (6, 2)$, and the optimal value is 28.
(d) Figure omitted. The optimal solution is $(x_1, x_2) = (0, 1)$, and the optimal value is 2.

p.21 Advanced

The figure is as follows.



From a different view point, we can see that plane $z = 3x_1 + 5x_2 + 4x_3$ is touching a vertex of the polyhedron.



The optimal solution is $(x_1, x_2, x_3) = (0, \frac{9}{4}, \frac{5}{8})$, and the optimal value is $\frac{55}{4}$.

※ For two variables, optimization can be done visually using a figure. With three variables, however, it becomes difficult. With even more variables, it becomes practically impossible.

p.29 Practice

(a) minimize $z = -4x_1 - 2x_2' + 2x_2''$

subject to

$$2x_1 + 2x_2' - 2x_2'' + s_1 = 4$$
$$3x_1 + 6x_2' - 6x_2'' - s_2 = 9$$
$$x_1, x_2', x_2'', s_1, s_2 \geq 0$$

※ To make $b_i \geq 0$, constraint $-3x_1 - 6x_2 \leq -9$ is transformed into $3x_1 + 6x_2 \geq 9$.

(b) minimize $z = -3x_1 - 5x_2 - 4x_3' + 4x_3''$

subject to

$$4x_1 + 2x_2 + 3x_3' - 3x_3'' + s_1 = 6$$
$$-3x_1 + 4x_2 - 5x_3' + 5x_3'' - s_2 = 2$$
$$x_1, x_2, x_3', x_3'', s_1, s_2 \geq 0$$

※ To make $b_i \geq 0$, constraint $3x_1 - 4x_2 + 5x_3 \leq -2$ is transformed into $-3x_1 + 4x_2 - 5x_3 \geq 2$.