Large-Scale Knowledge Processing

Optimization Techniques (2)
Practice Brief Solutions

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The column vectors of the matrix A are as follows:

$A_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $A_2 = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$, $A_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $A_4 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Since the rank of $(A_1 \ A_2) = \begin{pmatrix} 2 & 2 \\ 3 & 6 \end{pmatrix}$ is 2, the vectors A_1 and A_2 are linearly independent.

Solving $\begin{pmatrix} 2 & 2 \\ 3 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}$, we obtain $x_1 = \frac{4}{3}$ and $x_2 = \frac{2}{3}$.

Therefore, the basic solution with respect to the basis vectors A_1 and A_2 is $x = \begin{pmatrix} \frac{4}{3} \\ \frac{2}{3} \\ 0 \\ 0 \end{pmatrix}$.

※ Since x_1 and x_2 are nonnegative, this basic solution is a feasible basic solution.

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The basic solution corresponding to A_1 and A_2 is $\begin{pmatrix} \frac{4}{3} \\ \frac{2}{3} \\ 0 \\ 0 \end{pmatrix}$.

The basic solution corresponding to A_1 and A_3 is $\begin{pmatrix} \frac{8}{3} \\ 0 \\ -\frac{4}{3} \\ 0 \end{pmatrix}$.

The basic solution corresponding to A_1 and A_4 is $\begin{pmatrix} 2 \\ 0 \\ 0 \\ 2 \end{pmatrix}$.

The basic solution corresponding to A_2 and A_3 is $\begin{pmatrix} 0 \\ \frac{4}{3} \\ \frac{4}{3} \\ 0 \end{pmatrix}$.

The basic solution corresponding to A_2 and A_4 is $\begin{pmatrix} 0 \\ 2 \\ 0 \\ -4 \end{pmatrix}$.

The basic solution corresponding to A_3 and A_4 is $\begin{pmatrix} 0 \\ 0 \\ 4 \\ 8 \end{pmatrix}$.

Verify that $(x_1, x_2) = (\frac{4}{3}, \frac{2}{3})$, $(\frac{8}{3}, 0)$, $(2, 0)$, $(0, \frac{4}{3})$, $(0, 2)$, and $(0, 0)$ are the intersection points of the four lines $2x_1 + 2x_2 = 4$, $3x_1 + 6x_2 = 8$, $x_1 = 0$, and $x_2 = 0$. Also, verify that feasible basic solutions correspond to the vertices of the feasible region.

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We choose three linearly independent column vectors from A_1, A_2, A_3, A_4, and A_5 as basis vectors. There are $\binom{5}{3} = 10$ possible choices, and the corresponding basic solutions are as follows.

- Basic solution for A_1, A_2, A_3: $(\frac{4}{3}, \frac{2}{3}, 0, 0, 0)$
- Basic solution for A_1, A_2, A_4: $(\frac{4}{3}, \frac{2}{3}, 0, 0, 0)$
- Basic solution for A_1, A_2, A_5: $(\frac{4}{3}, \frac{2}{3}, 0, 0, 0)$
- Basic solution for A_1, A_3, A_4: $(4, 0, -4, -4, 0)$
- Basic solution for A_1, A_3, A_5: $(\frac{8}{3}, 0, -\frac{4}{3}, 0, \frac{4}{3})$
- Basic solution for A_1, A_4, A_5: $(2, 0, 0, 2, 2)$
- Basic solution for A_2, A_3, A_4: $(0, 1, 2, 2, 0)$
- Basic solution for A_2, A_3, A_5: $(0, \frac{4}{3}, \frac{4}{3}, 0, -\frac{4}{3})$
- Basic solution for A_2, A_4, A_5: $(0, 2, 0, -4, -4)$
- Basic solution for A_3, A_4, A_5: $(0, 0, 4, 8, 4)$

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Let x_2 be a basic variable and x_4 a nonbasic variable. Rewrite the equation for $x_4 = \dots$ into the form $x_2 = \dots$, and substitute it into the other equations.

$$\begin{aligned} z &= -9 - x_1 + x_4 \\ x_3 &= 1 - x_1 + \tfrac{1}{3}x_4 \\ x_2 &= \tfrac{3}{2} - \tfrac{1}{2}x_1 - \tfrac{1}{6}x_4 \end{aligned}$$