Large-Scale Knowledge Processing

Optimization Techniques (4) Supplementary Materials
Practice Brief Solutions

p.4 Practice

(a) The dual problem is given as follows.

$$\begin{array}{lll} \text{maximize} & w = & 9y_2 \\ \text{subject to} & & 5y_1 + 2y_2 \leq -5 \\ & & 2y_1 - 2y_2 \leq 1 \\ & & -y_1 + 2y_2 \leq 1 \\ & & 3y_1 + 2y_2 \leq -1 \\ & & y_1, y_2 : \text{free variables} \end{array}$$

Since the optimal solution of the primal problem is $(x_1, x_2, x_3, x_4) = (\frac{3}{4}, 0, \frac{15}{4}, 0)$, the complementary slackness theorem implies the following equalities.

$$\begin{aligned} 5y_1 + 2y_2 &= -5 \\ -y_1 + 2y_2 &= 1 \end{aligned}$$

Solving these equations, we obtain the optimal solution of the dual problem $(y_1, y_2) = (-1, 0)$, and the optimal value is 0.

※ By the strong duality theorem, the optimal value of the dual problem is equal to that of the primal problem.

※ By substituting the values of the optimal solution of the dual problem into the objective function, we can directly verify that the optimal values of the primal and dual problems are equal.

(b) The dual problem is given as follows.

$$\begin{array}{lll} \text{maximize} & w = & 8y_1 + 10y_2 \\ \text{subject to} & & 3y_1 + y_2 \leq 2 \\ & & -y_1 + 3y_2 \leq -3 \\ & & 2y_1 - 2y_2 \leq -2 \\ & & 3y_1 + 2y_2 \leq 5 \\ & & y_1, y_2 : \text{free variables} \end{array}$$

Since the optimal solution of the primal problem is $(x_1, x_2, x_3, x_4) = (0, 9, \frac{17}{2}, 0)$, the complementary slackness theorem implies the following equalities.

$$\begin{aligned} -y_1 + 3y_2 &= -3 \\ 2y_1 - 2y_2 &= -2 \end{aligned}$$

Solving these equations, we obtain the optimal solution of the dual problem $(y_1, y_2) = (-3, -2)$, and the optimal value is -44.

※ Even if the primal problem is not in standard form, the dual problem can still be formulated.