Large-Scale Knowledge Processing

Optimization Techniques (4)
Practice Brief Solutions

p.7 Practice

(a) The dual problem is given as follows.

$$\begin{array}{ll} \text{minimize} & z = 7y_1 + 8y_2 + 9y_3 \\ \text{subject to} & y_1 \qquad + \ y_3 \geq 4 \\ & 2y_1 + \ y_2 \qquad \geq 3 \\ & \qquad 2y_2 + \ y_3 = 2 \\ & y_1 : \text{free variable}, \ y_2, y_3 \geq 0 \end{array}$$

(b) The dual problem is given as follows.

$$\begin{array}{ll} \text{maximize} & z = 7y_1 + 8y_2 + 9y_3 \\ \text{subject to} & y_1 \qquad + \ y_3 \leq 4 \\ & 2y_1 + \ y_2 \qquad \leq 3 \\ & \qquad 2y_2 + \ y_3 = 2 \\ & y_1 : \text{free variable}, \ y_2, y_3 \geq 0 \end{array}$$

p.8 Practice

(1-a) The standard form is given as follows.

$$\begin{array}{ll} \text{minimize} & z = -x_1 - 2x_2 \\ \text{subject to} & -x_1 - x_2 + x_3 = 1 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

Therefore, the dual problem is given as follows.

$$\begin{array}{ll} \text{maximize} & w = y_1 \\ \text{subject to} & -y_1 \leq -1 \\ & -y_1 \leq -2 \\ & y_1 \leq 0 \\ & y_1 : \text{free variable} \end{array}$$

(1-b) The standard form is given as follows.

$$\begin{array}{ll} \text{minimize} & z = -x_1 - 2x_2 \\ \text{subject to} & -x_1 - x_2 - x_3 = 1 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

Therefore, the dual problem is given as follows.

$$\begin{array}{ll} \text{maximize} & w = y_1 \\ \text{subject to} & -y_1 \leq -1 \\ & -y_1 \leq -2 \\ & -y_1 \leq 0 \\ & y_1 : \text{free variable} \end{array}$$

(1-c) The standard form is given as follows.

$$\begin{array}{ll} \text{minimize} & z = -4x_1 - 2x_2' + 2x_2'' \\ \text{subject to} & 2x_1 + 2x_2' - 2x_2'' + s_1 = 4 \\ & 3x_1 + 6x_2' - 6x_2'' - s_2 = 9 \\ & x_1, x_2', x_2'', s_1, s_2 \geq 0 \end{array}$$

Therefore, the dual problem is given as follows.

$$\begin{array}{lll}
\text{maximize} & w = & 4y_1 + 9y_2 \\
\text{subject to} & & 2y_1 + 3y_2 \le -4 \\
& & 2y_1 + 6y_2 \le -2 \\
& & -2y_1 - 6y_2 \le 2 \\
& & y_1 \le 0 \\
& & -y_2 \le 0 \\
& & y_1, y_2 : \text{free variables}
\end{array}$$

(1-d) The standard form is given as follows.

$$\begin{array}{lll}
\text{minimize} & z = -3x_1 - 5x_2 - 4x_3' + 4x_3'' & \\
\text{subject to} & 4x_1 + 2x_2 + 3x_3' - 3x_3'' + s_1 = 6 & \\
& -3x_1 + 4x_2 - 5x_3' + 5x_3'' - s_2 = 2 & \\
& x_1, x_2, x_3', x_3'', s_1, s_2 \ge 0 &
\end{array}$$

Therefore, the dual problem is given as follows.

$$\begin{array}{lll}
\text{maximize} & w = & 6y_1 + 2y_2 \\
\text{subject to} & & 4y_1 - 3y_2 \le -3 \\
& & 2y_1 + 4y_2 \le -5 \\
& & 3y_1 - 5y_2 \le -4 \\
& & -3y_1 + 5y_2 \le 4 \\
& & y_1 \le 0 \\
& & -y_2 \le 0 \\
& & y_1, y_2 : \text{free variables}
\end{array}$$

(1-e) The standard form is given as follows.

$$\begin{array}{ll}
\text{minimize} & z = -4x_1 - 5x_2 \\
\text{subject to} & 2x_1 + 2x_2 + x_3 = 4 \\
& 3x_1 + 6x_2 + x_4 = 8 \\
& x_1 + 4x_2 + x_5 = 4 \\
& x_1, x_2, x_3, x_4, x_5 \ge 0
\end{array}$$

Therefore, the dual problem is given as follows.

$$\begin{array}{ll}
\text{maximize} & w = 4y_1 + 8y_2 + 4y_3 \\
\text{subject to} & 2y_1 + 3y_2 + y_3 \le -4 \\
& 2y_1 + 6y_2 + 4y_3 \le -5 \\
& y_1 \le 0 \\
& y_2 \le 0 \\
& y_3 \le 0 \\
& y_1, y_2, y_3 : \text{free variables}
\end{array}$$

(1-f) The standard form is given as follows.

$$\begin{array}{lll}
\text{minimize} & z = & -4x_1 - 2x_2 \\
\text{subject to} & & x_1 + x_2 + x_3 = 8 \\
& & x_1 + x_4 = 6 \\
& & x_1 + 2x_2 - x_5 = 2 \\
& & x_1, x_2, x_3, x_4, x_5 \ge 0
\end{array}$$

Therefore, the dual problem is given as follows.

$$
\begin{array}{ll}
\text{maximize} & w = 8y_1 + 6y_2 + 2y_3 \\
\text{subject to} & y_1 + y_2 + y_3 \le -4 \\
& y_1 + 2y_3 \le -2 \\
& y_1 \le 0 \\
& y_2 \le 0 \\
& - y_3 \le 0 \\
& y_1, y_2, y_3 : \text{free variables}
\end{array}
$$

(2) See p. 6 of the slide.

p.15 Practice

The standard form of this problem is given as follows.

$$
\begin{array}{ll}
\text{minimize} & z = -x_1 - 2x_2 \\
\text{subject to} & x_1 + x_2 - x_3 = 1 \\
& x_1, x_2, x_3 \ge 0
\end{array}
$$

The dual problem of this standard form is given as follows.

$$
\begin{array}{ll}
\text{maximize} & w = y_1 \\
\text{subject to} & y_1 \le -1 \\
& y_1 \le -2 \\
& -y_1 \le 0 \\
& y_1 : \text{free variable}
\end{array}
$$

There exists no value of y_1 that satisfies all three constraints simultaneously. Hence, the dual problem is infeasible.

※ The infeasibility of the dual problem can also be confirmed by converting the dual problem into its standard form and applying the two-phase method.

p.15 Practice

(a) The dual problem of this standard form is given as follows.

$$
\begin{array}{ll}
\text{minimize} & z = -x_1 - 2x_2 \\
\text{subject to} & -x_1 - x_2 + x_3 = 1 \\
& x_1, x_2, x_3 \ge 0
\end{array}
$$

The dual problem of this standard form is given as follows.

$$
\begin{array}{ll}
\text{maximize} & w = y_1 \\
\text{subject to} & -y_1 \le -1 \\
& -y_1 \le -2 \\
& y_1 \le 0 \\
& y_1 : \text{free variable}
\end{array}
$$

Since there is no value of y_1 that satisfies all three constraints, the dual problem is infeasible.

※ The primal problem is unbounded, whereas the dual problem is infeasible.

(b) The standard form of this problem is given as follows.

$$
\begin{array}{ll}
\text{minimize} & z = -x_1 - 2x_2 \\
\text{subject to} & -x_1 - x_2 - x_3 = 1 \\
& x_1, x_2, x_3 \ge 0
\end{array}
$$

The dual problem of this standard form is given as follows.

$$
\begin{array}{lll}
\text{maximize} & w = & y_1 \\
\text{subject to} & & -\,y_1 \leq -1 \\
& & -\,y_1 \leq -2 \\
& & -\,y_1 \leq 0 \\
& & y_1 : \text{free variable}
\end{array}
$$

Since the three constraints are satisfied for any $y_1 \geq 2$, the objective value can be made arbitrarily large. Hence, the dual problem is unbounded.

※ The primal problem is infeasible, whereas the dual problem is unbounded.